RCA RADIOTRON COMPANY, INC.
HARRISON (RCA) NEW JERSEY

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UNIFIED SALES--ENGINEERING SERVICE
TO
EQUIPMENT MANUFACTURERS

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APPLICATION NOTE No. 36
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APPLICATION NOTES

APPLICATION NOTE ON LISSAJOU'S FIGURES

When varying voltages are applied to the deflecting plates of a cathode-ray tube, a pattern is obtained on the fluorescent screen. The shape of this pattern depends upon the wave forms of the applied voltages and upon their phase relationships. In this Application Note a study of these patterns, or Lissajou's figures, will be made with particular attention to their development, their use in identifying frequency ratios, and the effect of phase shift.

Simple Figures

Figure 1 represents a sine-wave voltage (A) applied to the vertical pair of deflecting plates of a cathode-ray tube and an identical voltage (B) applied to the horizontal deflecting plates. The resulting pattern, shown by (C) is a straight line having a 45 degree slope. The direction of the slope of this line is determined by the phase relation of the two voltages as illustrated in Figures 6A and 6E.* Figure 2 illustrates the case of two identical voltages having the same amplitude but 90°, or 270°, out of phase. In this case, the resulting figure is a circle. If one of the figures is of greater amplitude than the other, the resulting figure will be an ellipse as shown by Figure 6C. If the phase relation is such that one voltage leads by 45°, or 315°, the resulting pattern will be that of Figure 6D; if leading by 135°, or 225°, the resulting pattern will be that of 6B. Figures 1 to 5 inclusive, show a graphical method for determining the resulting pattern, where the wave shapes, the relative amplitudes, the phase relation, and the frequencies of the two deflecting voltages are known. By means of the cathode-ray tube, the resultant pattern is traced on the fluorescent screen. Con-

*Figures 6 to 12 inclusive, and Figures 22 and 23 adapted from "Frequency Measurements with the Cathode Ray Oscillograph," Fredrick J. Rasmussen, A. I. E. E. Transactions, November, 1926, Vol. XLV, Pages 1256-65.

versely, from this pattern, the frequency, and the phase relations of the two deflecting voltages can be determined. Where, in addition, the wave form is known for one of the deflecting voltages, the wave form of the other can readily be obtained by graphical analysis.

Figures 1, 2, and 6A to 6E are for a 1 : 1 frequency ratio. When a 2 : 1 ratio of the voltages applied to the deflecting plates is the case, the wave shapes of Figures 6A to 6E become those shown by Figures 6F to 6J.

Complex Figures

As the ratio of the frequencies increases, the pattern becomes more complex. In Figure 3, A and B are the voltages applied to the deflecting plates. In this case, the frequency of A is three times that of B. The resultant figure (C) shows a 1 : 3 pattern in which both voltages start in phase. Figure 4 is the same as Figure 3 except that voltage A is started 90° out of phase with respect to voltage B. Figure 5C shows the resultant pattern obtained where B is a saw-tooth wave and A a sine wave. This is of interest because this type of wave form results when a linear timing axis is used. Figures 7, 8, and 9 show patterns of increasing complexity, Figure 9 being an 8 : 6 pattern.

Determination of Frequency by Inspection of Pattern

When the cathode-ray oscillograph is used for calibration purposes, frequency ratios of less than 10 : 1 can be readily determined by visual inspection of the image. For frequency ratios greater than 10 : 1, the complexity of the pattern makes visual determination difficult and requires determination by means of photograph. In general, the standard frequency selected should be one whose multiples and submultiples will cover the desired range and provide the simplest patterns.

In examining Lissajou's figures, one should consider them as the side view or elevation of a picture traced on a glass cylinder on which the observer may view the wave as it travels around the cylinder. The illusion is clearest when the whole figure rotates slowly. Figure 10 is a simple single-line pattern having a frequency ratio of 6 : 1. With a base frequency of 60 cycles this pattern would be the picture for 360 cycles, or with a base frequency of 100 cycles, would be the picture of 600 cycles. The frequency ratio is determined by counting the peaks (six in number) of the waves in the horizontal plane and the number of end loops which for this case is one; hence, a frequency ratio of 6 : 1. In Figure 10, the front tracing has been made heavy and the back tracing light so that the two can be readily distinguished. If the figure were to be shifted slightly, the front and back waves might appear to be one. This condition might mislead the observer to believe that the frequency ratio was less than 6 : 1. Adjustment of the unknown frequency so that the pattern rotates very slowly, or stands still with the rear peaks uncovered by the front peaks, will make determination simplest. It will be observed that the wave form of Figure 10 corresponds to that of Figure 13*, a single-line pattern whose back trace is not visible. Figure 19 shows the simplest 2 : 1 wave or two-line figure. Figure 11 is a complete two-line figure illustrating a ratio of 9 : 2, which again, is readily determined by counting the number of peaks along the top of the figure and the number of loops at the end. Figure 12 has 16 peaks and is a three-line pattern, indicating a frequency ratio of 16 : 3.

Figures 10, 11, and 12 illustrate patterns as they generally appear on the fluorescent screen. Figures 7 to 9, and 13 to 19 are shown as pictures whose appearance suggests that the pattern has been developed on a plane. They have been shown in this fashion to facilitate study.

An optional method for determination of the frequency ratio is that of comparing the number of peaks on a given figure with the horizontal lines of intersections on the figure instead of with the number of loops at the end of the figure. A study of some of the patterns will make this clear. In Figure 16, there is a single line of intersections along the axes of the figures. It can easily be seen that this is a two-line figure by comparing it with the single-line Figures 13 and 19. Figures 12, 15, and 17 having two horizontal lines of intersections each spaced approximately one-third from the top and bottom are three-line patterns. In the same manner, the four-line patterns of Figures 7, 14, and 18 are distinguished by three lines of intersection, the five-line pattern of Figure 8 by four lines of intersection, and the six-line pattern of Figure 9 by five lines of intersection with characteristic positions for these lines in each case. Thus, the frequency ratio is also equal to the number of peaks on circumference divided by the term, one + number of horizontal lines of intersection.

<u>Sequence of Patterns</u>

Of the patterns from 1 to 19, those of Figures 13, 19, and 3 show simple ratios of 1 : 1, 2 : 1, and 3 : 1. Both these direct multiples and fractional multiples of the base frequency are available to the user of the cathode-ray oscillograph. For example, with a base frequency of 60 cycles, the following tabulation will serve to illustrate the sequence of relatively simple patterns obtained as the frequency of the variable unit is decreased from a 1 : 1 ratio of frequencies to a 3 : 1 ratio.

*Figures 13 to 19 inclusive, and Figures 20 and 21 adapted from "The Cathode Ray Oscillograph in Radio Research," R. A. Watson Watt. Published by His Majesty's Stationery Office, London, England.

Frequency In Cycles/Sec.	Frequency Ratio* Whole Number	Fractional	Illustrated By Figure
60	1 : 1	1 : 1	13
75	5 : 4	1¼ : 1	14
80	4 : 3	1-1/3 : 1	15
90	3 : 2	1½ : 1	16
100	5 : 3	1-2/3 : 1	17
105	7 : 4	1¾ : 1	18
120	2 : 1	2 : 1	19
135	9 : 4	2¼ : 1	-
140	7 : 3	2-1/3 : 1	-
150	5 : 2	2½ : 1	-
160	8 : 3	2-2/3 : 1	-
165	11 : 4	2¾ : 1	-
180	3 : 1	3 : 1	3

*The frequency ratio is expressed either as a ratio of two integers, the first of which represents the number of peaks and the second the number of lines in the patterns, or as a ratio of a whole number and a fraction to unity. The denominator of the fraction is equal to the number of lines in the figure.

If the base frequency is 1000 cycles instead of 60, the same ratios hold. Thus, instead of 60 to 180 cycles, the frequencies for these patterns would be those for 1000 to 3000 cycles with intermediate values of 1250, 1333-1/3, 1500, 1666-2/3, 1750 cycles, 2000 cycles, 2250 cycles, 2333-1/3 cycles, 2500 cycles, 2666-2/3 cycles, and 2750 cycles.

Elliptical and Circular Figures

When waves having frequency ratios greater than 10 : 1 are compared, accurate determinations may be difficult with the front and back portions of the figures in the same horizontal plane. To separate the back and front portions, the figures can be displaced to show either on an ellipse or a circle.

For an ellipse, a phase-splitting device consisting of a resistance and a capacity is employed, see Figure 24. Resistance (R) is connected across one set of deflecting plates and capacitance (C) is connected across the other pair. Figures 20 and 21 show the same single-line pattern and were obtained by adjusting the circuit of Figure 24 for different vertical amplitudes. Figure 22 is a two-line pattern having a frequency ratio of 31 : 2. The frequency ratio of this figure would be much more difficult to determine without displacement.

To produce the type of pattern shown in Figure 23, a circular axis is developed using the circuit of Figure 24, with the exception that the voltage under study is introduced in series with anode No.2. It will be found that the peaks on this type of pattern will be somewhat blurred due to the defocusing effect caused by introduction of the voltage under study into the anode No.2 circuit. Defocusing can be minimized by keeping this voltage at a low amplitude.

Simplification of the Pattern

It was pointed out that the patterns of Figures 7 to 9, and 13 to 19 are developed on a plane. The resulting patterns are much simpler than they would be with their normal appearance because the back wave has been removed by spreading it out in the same plane with the front wave. The advantages of this simplified appearance can be obtained in practice by total elimination of the back wave. Where there is some doubt as to the number of lines in a pattern because of the presence of the additional lines of intersections observed in the back wave, this method will be of considerable assistance. Figure 11, for instance, is a two-line pattern, as is shown by the two loops at the end of the figure. However, because of the shift of the figure, the intersection made by the lines of the back wave with the lines of the front wave give it the same appearance as the five-line pattern of Figure 8. To eliminate the back wave, voltage of the same frequency as that used for the spreader, but 90° out of phase is applied to the control grid of the cathode-ray tube. Adjustment of this voltage will permit weakening the back wave and brightening the front wave, or the total elimination of the back wave.

LISSAJOU'S FIGURES

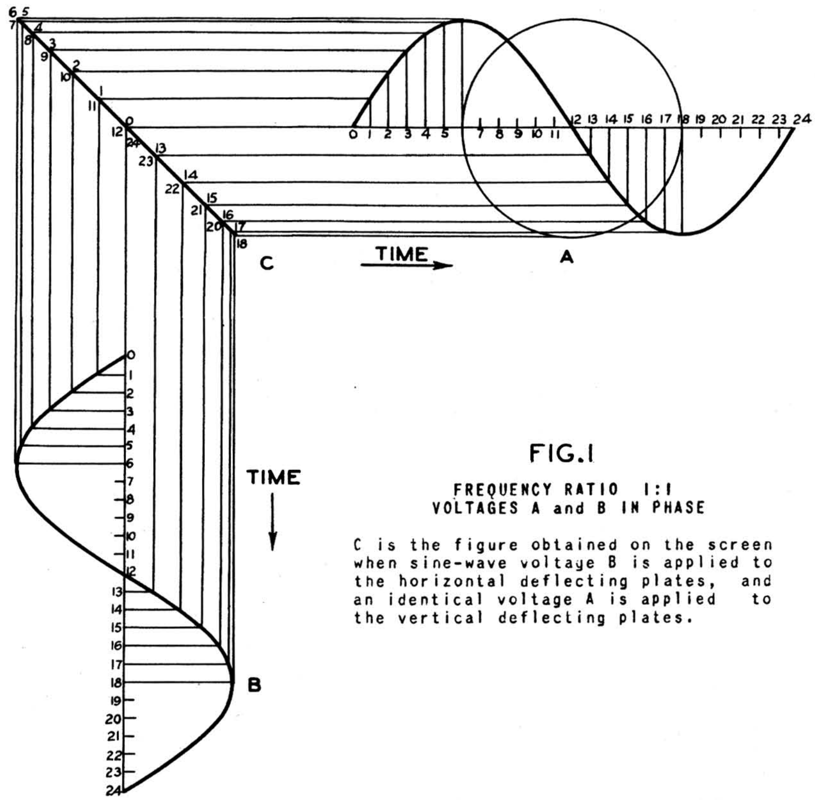

FIG.1

FREQUENCY RATIO 1:1
VOLTAGES A and B IN PHASE

C is the figure obtained on the screen when sine-wave voltage B is applied to the horizontal deflecting plates, and an identical voltage A is applied to the vertical deflecting plates.

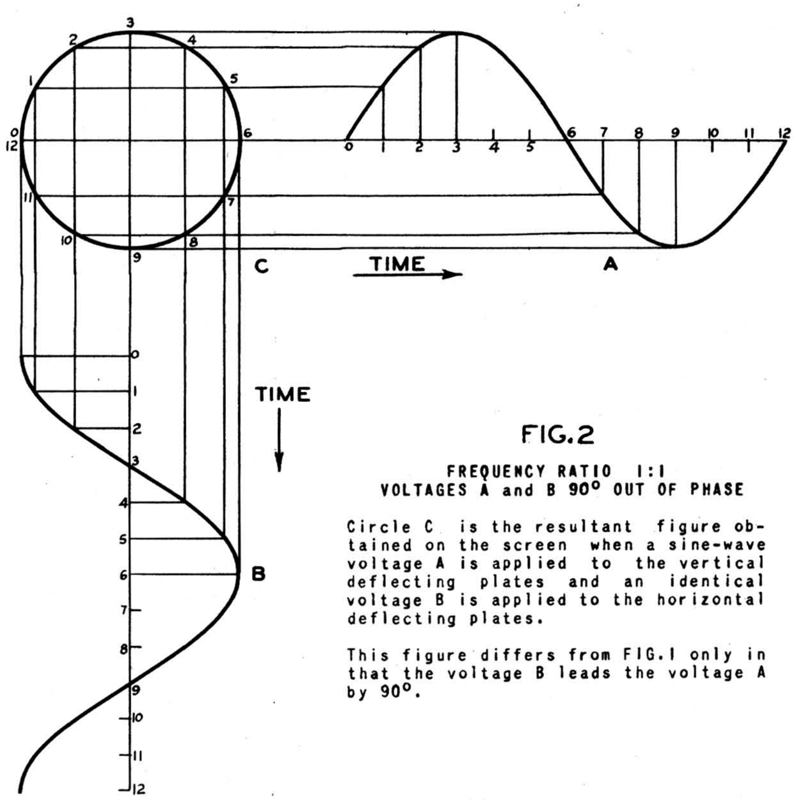

FIG.2

FREQUENCY RATIO 1:1
VOLTAGES A and B 90° OUT OF PHASE

Circle C is the resultant figure obtained on the screen when a sine-wave voltage A is applied to the vertical deflecting plates and an identical voltage B is applied to the horizontal deflecting plates.

This figure differs from FIG.1 only in that the voltage B leads the voltage A by 90°.

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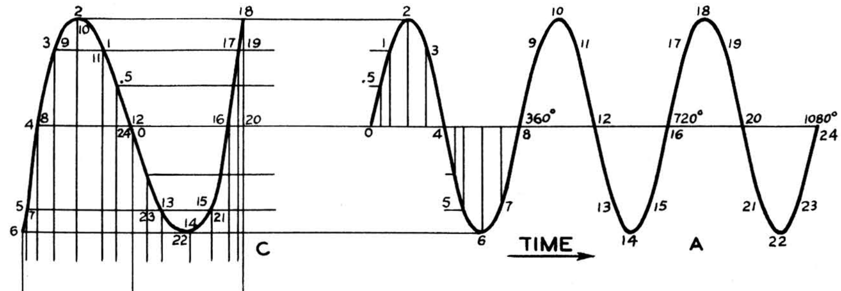
LISSAJOU'S FIGURES



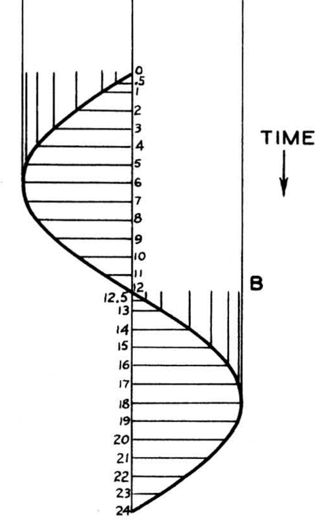
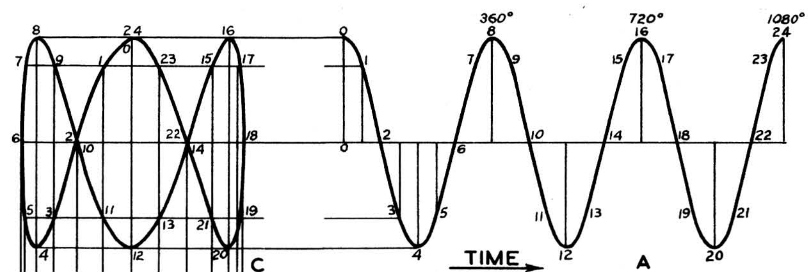


FIG.3

FREQUENCY RATIO 1:3
VOLTAGES A and B IN PHASE

Pattern C is the resultant figure when a voltage B is applied to the horizontal deflecting plates and a voltage A whose frequency is three times that of B, is applied to the vertical deflecting plates.

In the pattern C, peak 10 is directly behind peak 2 and peak 18 is at the right.



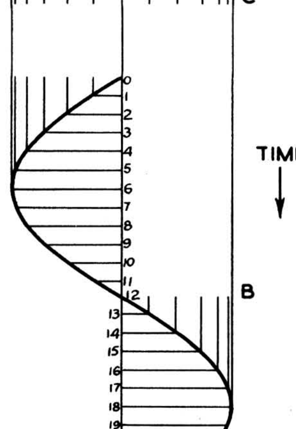


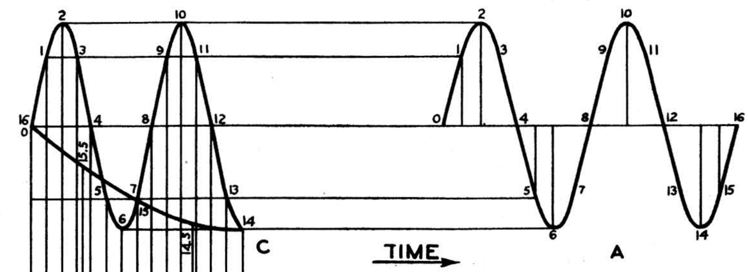
FIG.4

FREQUENCY RATIO 1:3
VOLTAGES A and B 90° OUT OF PHASE

FIG.4 shows the effect of phase shift on the pattern of FIG.3.

The voltage A is three times the frequency of the voltage B and leads it by 90°.

In this pattern, all three peaks are visible.



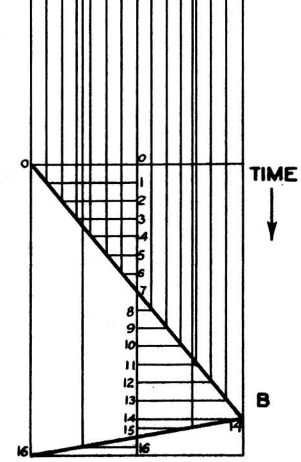


FIG.5

FREQUENCY RATIO 1:2
VOLTAGES A and B IN PHASE

FIG.5 shows a sine-wave voltage A applied to the vertical deflecting plates and a saw-tooth wave B applied to the horizontal deflecting plates.

Wave B is linear from 0 to 14: hence, on the pattern C, the sine wave A appears undistorted. During the interval 14 to 16, the trace returns to the starting point 16.

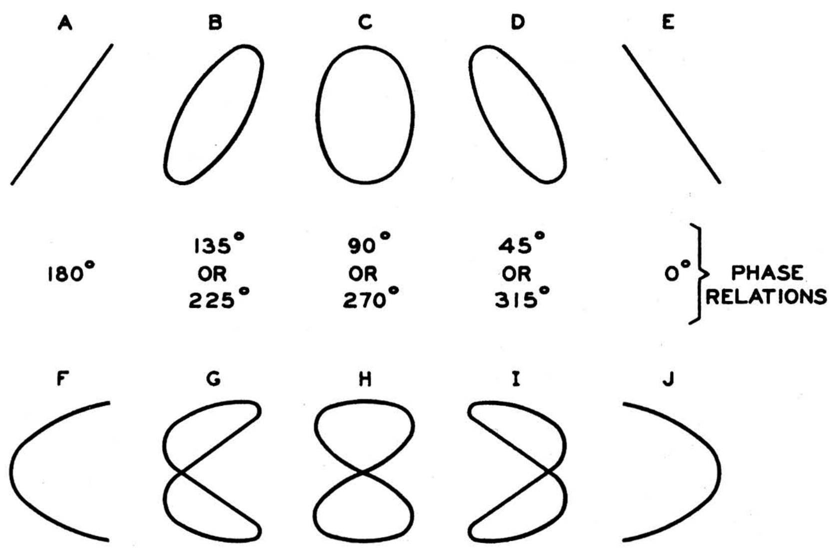


FIG. 6 HORIZONTAL TIMING AXIS→
VERTICAL TIMING AXIS↓

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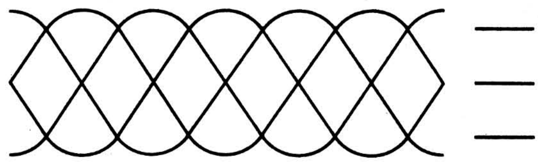
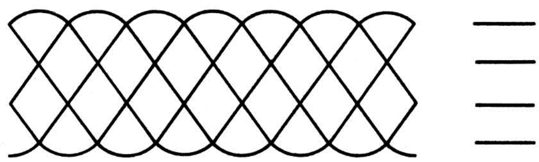
LISSAJOU'S FIGURES



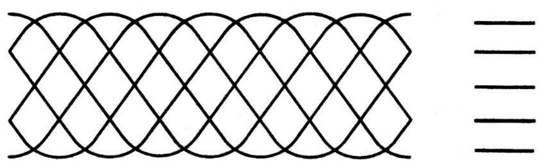
6 : 4 $1\frac{2}{4} : 1$

FIG. 7



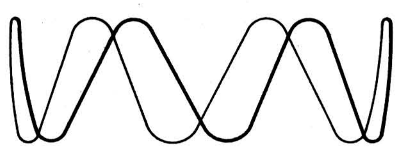
7 : 5 $1\frac{2}{5} : 1$

FIG. 8



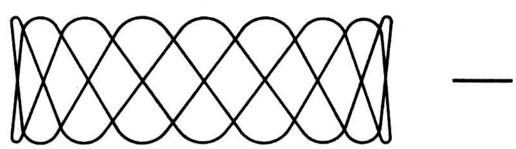
8 : 6 $1\frac{2}{6} : 1$

FIG. 9



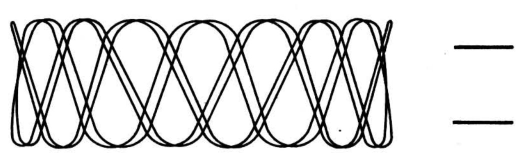
6 : 1

FIG. 10



9 : 2 $4\frac{1}{2} : 1$

FIG. 11



16 : 3 $5\frac{1}{3} : 1$

FIG. 12

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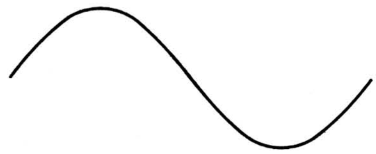
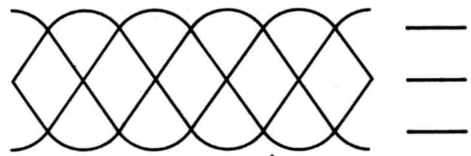
LISSAJOU'S FIGURES



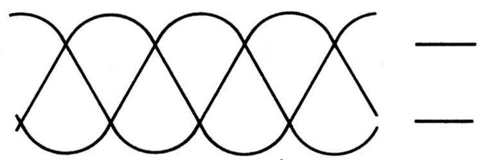
1:1

FIG. 13



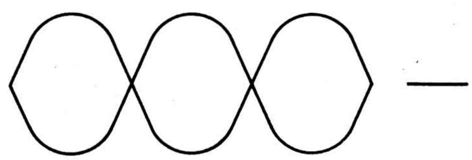
5:4 $1\frac{1}{4}:1$

FIG. 14



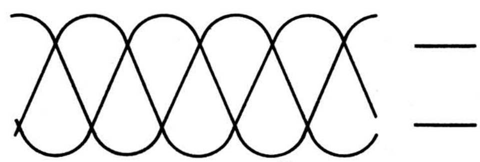
4:3 $1\frac{1}{3}:1$

FIG. 15



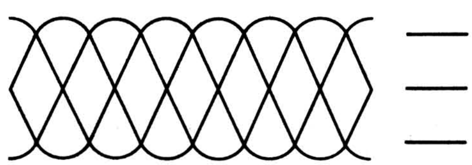
3:2 $1\frac{1}{2}:1$

FIG. 16



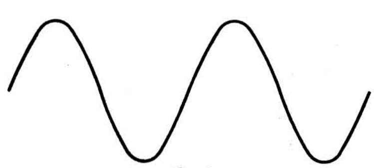
5:3 $1\frac{2}{3}:1$

FIG. 17



7:4 $1\frac{3}{4}:1$

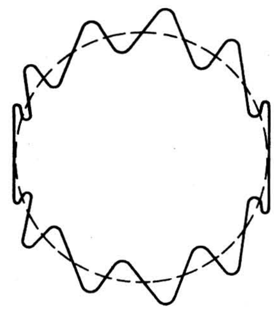
FIG. 18



2:1

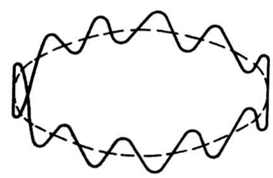
FIG. 19

LISSAJOU'S FIGURES



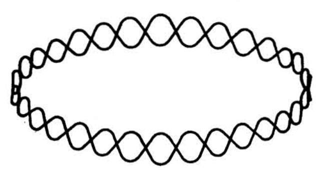
10 : 1

FIG. 20



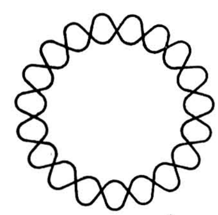
10 : 1

FIG. 21



31:2 15½ : 1

FIG. 22



19:2 9½ : 1

FIG. 23

PHASE-SPLITTING CIRCUIT FOR OBTAINING ELLIPTICAL OR CIRCULAR AXIS

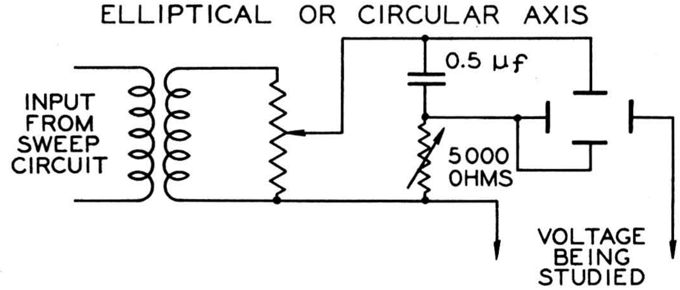


FIG. 24

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